The Isgur Wise Function

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***Abstract*— The Isgur Wise function is basically a non perturbative character, which has in recent years been studied within various quark models, besides QCD rules, the skyrme model, the MIT Bag model and the dispersion approach. The exact form of the Isgur Wise functions is not known however though a vital component seems to be the hardon wave functions themselves are an important topic in QCD. It is therefore meaningful to test any specific QCD inspired model by calculating the Isgur Wise function with it and to study its phenomenology. In this paper we study the Isgur Wise function within various quark models like QCD sum rule, skyrme model, MIT big model, Dispersion approach etc. and summaries it.**

***Keywords*— Isgur Wise function, Hardon, Theory, Function etc.**

I. INTRODUCTION

In recent years, considerable experimental and theoretical efforts have been undertaken to understand the physics of hard on containing a heavy quark. It is also known that in the limit of infinity large quark mass, additional symmetries beyond the ones of QCD arises, which enables one to obtain model independent information on the weak decay matrix elements of heavy mesons. In this limit, all heavy quark bilinear current matrix elements are described in terms of one form factor the so called Isgur Wise function in leading order.

II. THE ISGUR WISE FUNCTION

Isgur, Wise, Georgi and others showed that in weak semi-leptonic decays of B mesons to D or $D*$ mesons, all the form factors that describe these decays are expressible in terms of a single universal function of velocity transfer, which is normalized to unity at zero-recoil. This function is known as the Isgur-Wise function. It measures the overlap of the wave functions of the light degrees of freedom in the initial and final mesons moving with velocities v and v' respectively. The Isgur-Wise functions are defined as $\xi(y)$, where $y = v.v'$ and $\xi(y = 1) = 1$ is the normalization condition at the zero-recoil point $(v = v')$. The condition $\xi(y = 1) = 1$ is a consequence of the conservation of vector current and implies the complete overlap of the wave functions if $v = v'$.

The implications of Heavy Quark Symmetry for the semi-leptonic decays of B mesons, $B \to D * l\mu$ and $B \to Dl\mu$ are that, if we assume that b and c quarks are heavy enough to satisfy the requirements of High Quark Effective Theory (HQET), then the six real, generally independent form factors that describe these decays are expressible in terms of a single universal function of velocity transfer.

In HQET, the velocity of a heavy quark is conserved with respect to light degrees of freedom and the mass dependent part of the momentum operator is removed by redefining a field $h_Q(v,x)$ which annihilates a heavy quark Q with velocity v as

$$h_Q(v,x) = \frac{(1+v)}{2}\exp\left(i\, m_Q v.x\right) Q(x)$$

The new field carries the residual momentum $k_\mu = P_\mu - m_Q v_\mu$ of order Λ_{QCD}. The effective Lagrangian for strong interactions in the limit $m_Q \to \infty$ then becomes

$$\mathcal{L}_{eff} = \bar{h}_Q iv.Dh_Q - \delta m_Q \bar{h}_Q h_Q$$

Where D_μ is the covariant derivative and δm_Q denotes the residual mass of the heavy quark in the effective theory. But m_Q is not uniquely defined. Moreover, HQET will be consistent only if δm_Q and k_μ are finite in the limit $m_Q \to \infty$. There is a unique choice m_Q^* such that $\delta m_Q = 0$ which provides a non perturbative definition of the heavy quark mass and is most commonly used in HQET. At leading order in the heavy quark expansion, for a transition between two heavy mesons induced by a weak current, $M \to M'$, the matrix element factorizes into a kinematical part depending on the mass and spin-parity quantum numbers of the mesons and a reduced matrix element describing the elastic transition of the light degrees of freedom. For spin wave functions of the form

$$\mathcal{M}(v) = \sqrt{m_M}\frac{(1+v)}{2}\begin{cases}-\gamma_5 & ; \quad J^P = 0^-, \\ \epsilon.\gamma & ; \quad J^P = 1^-,\end{cases} \qquad (1.2)$$

$$< M'|\bar{h}_Q \Gamma h_Q|M >= -\xi(y) Tr\{\overline{\mathcal{M}'}(v')\Gamma\mathcal{M}(v)\} \quad (1.2)$$

Where $y = v.v'$ and $\xi(y)$ is the universal Isgur Wise function. For the semi leptonic decays of B mesons, a set of form factors $h_i(y)$ can be defined such that

$$< D(v')|V_\mu|\bar{B}(v) >= \sqrt{m_B m_D}[h_+(y)(v+v')_\mu + h_-(y)(v-v')],$$

$$< D^*(v')|V_\mu|\bar{B}(v) >= \sqrt{m_B m_{D^*}} h_v(y)\epsilon_{\mu v\alpha\beta}\,\epsilon^{*v}v'^{\alpha}v^{\beta},$$

$$< D^*(v')|A_\mu|\bar{B}(v) >= \sqrt{m_B m_{D^*}}[h_{A1}(y)(y+1)\epsilon^*_\mu - h_{A2}(y)\epsilon^*.vv_\mu - h_{A3}(y)\epsilon^*.vv'_\mu] \quad (1.3)$$

Where $V_\mu = \bar{c}\gamma_m b$ and $A_\mu = \bar{c}\gamma_\mu\gamma_5 b$, and ϵ_μ is the polarization of the D^* meson. In the heavy quark infinite mass limit, (1.3) leads to

$$h_+(y) = h_V(y) = h_{A1}(y) = h_{A3}(y) = \xi(y)$$

$$h_-(y) = h_{A2}(y) = 0 \quad (1.4)$$

Thus, at leading order in $\frac{\bar{\Lambda}}{m_Q^*}$, the HQET predictions for form factors in $B \to D^* l\mu$ decay involve one known function only, the Isgur Wise function $-\xi(y)$. Similarly, for the transition in which a heavy quark c is changed to a light quark s, only 2 form factors are needed. The corresponding theoretical description of decays of heavy hadrons is thus greatly simplified and model dependence is greatly reduced. Further, such reduction in the number of form factors leads to possible experimental predictions.

III. Conclusion

The Isgur Wise function is however not calculable in perturbative QCD. It is calculable in principle, using non-perturbative techniques such as lattice gauge theory QCD sum rules. The mass parameter $\bar{\Lambda}$ appears at leading order of the heavy-quark expansion and cannot be calculated perturbatively. Moreover, the heavy quark symmetry is an approximate symmetry and correction arises since the quark masses are not infinite. Corrections to the limit $m_Q \to \infty$ may be studied systematically in the framework of HQET. The leading symmetry-breaking corrections are from terms of order $\frac{1}{m_Q^*}$ and from the QCD corrections due to the coupling of hard gluons to the heavy quarks. The corrections are given as power series expansions in two small parameters – (i) α_S taken at the scale of the heavy quark (QCD corrections), ie. $\alpha_S(m_Q^*)$ and (ii) the parameter $\frac{\bar{\Lambda}}{m_Q^*}$ where $\bar{\Lambda}$ is a scale of the light QCD degrees of freedom; ie. $\bar{\Lambda} \sim m_{hadron} - m_Q^*$. Thus

$$h_i(y) = [\alpha_i + \beta_i(y) + \gamma_i(y) + \cdots]\xi(y) \quad (1.5)$$

Where $\alpha_+ = \alpha_V = \alpha_{A1} = \alpha_{A3} = 1, \alpha_- = \alpha_{A2} = 0$ and $\beta_i(y)$ are the short-distance perturbative QCD corrections depending on $y, \alpha_S(m_Q^*)$ and the mass ratio of the quarks while $\gamma_i(\mathrm{y})$ contain the $\frac{1}{m_c^*}$ and $\frac{1}{m_b^*}$ corrections. Hence a calculation in HQET to a given order in $\frac{\bar{\Lambda}}{m_Q^*}$ and α_S is complete upto corrections that are higher order in these parameters. Although the relative corrections can be calculated using perturbative QCD, the m_Q^* corrections induce new uncalculable functions. For $B \to D^* l\mu$ decay there are four such uncalculabe functions. Hence the predictive power of the theory is reduced.

This is where phenomenological models again become important. It was thought that the discovery of Heavy Quark Symmetry would lead to model independent predictions and results, but HQET cannot predict many things. It may however show that some of the old model independent results were wrong. As an example, HQET predicts the normalization of the Isgur Wise function at zero recoil point. But, since HQET does not predict the shape of the Isgur Wise function, it makes sense to calculate the Isgur Wise function using a specific model. As HQET predicts the formalization of the Isgur Wise function, so a good model must give the same in the appropriate limit. In doing so, we can gain insight into HQET, i.e. Find a reasonable candidate Isgur Wise function. We can also gain insight into our model, i.e. See if our model properly obeys the normalization condition on the Isgur Wise function in the appropriate limit.

References

[1] N. Isgur and G. Karl. Phys. Lett., B72 : 109, (1977)
[2] N. Isgur and G. Karl. Phys. Lett., B72 : 353, (1978)
[3] N. Isgur and G. Karl. Phys. Rev., D18 : 4187, (1978)

[4] N. Isgur and G. Karl. Phys. Rev., D19 : 2653, (1979)
[5] N. Isgur and G. Karl. Phys. Rev., D20 : 1191, (1979)
[6] N. Isgur and M. B. Wise. Phys. Lett, B232 : 113, (1989)
[7] N. Isgur and M. B. Wise. Phys. Lett, B237 : 527, (1990)
[8] S. Godfrey and N. Isgur., Phys. Rev., D32 : 189, (1985)
[9] N. Isgur Proceedings of XVI International school of sub-nuclear Phys. Erica, 1978. Page 107, (1980).