Delay Analysis for Maximal Scheduling Algorithms to Minimize the Queue Overflow Probability

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***Abstract*--Link scheduling is an important functionality in wireless networks due to both the shared nature of the wireless medium and the variations of the wireless channel over time. Scheduling algorithms have focused on optimizing the long term average throughput of the users. The goal is to find algorithms for scheduling the transmissions such that the queues are stabilized at given offered loads. α – algorithm is a class of scheduling algorithm that is used to minimize the queue overflow probability. α is a parameter that takes values from the set of natural numbers. In this paper, to circumvent the difficulty of the multidimensional CoV problem, use lyapunov function to map the multidimensional CoV problem to a one dimensional problem, which allows us to bound the minimum cost to overflow. The advantage of working with the α – algorithm instead of exponential rule is that the α – algorithms are scale invariant.**

***Keywords*--Asymptotically Optimal Algorithms, Cellular System, Large Deviations, Queue Overflow Probability, Wireless Scheduling.**

I. Introduction

Scheduling is an important functionality in wireless networks due to both the shared nature of the wireless medium and the variations of the wireless channel over time. To focus on wireless scheduling algorithm for a downlink of a single cell that can minimize the queue overflow probability. In the past, it has been demonstrated that, by carefully choosing the scheduling decisions based on the channel state and / or the demand of the users, the system performance can be substantially improved (see, e.g. the references in [2]).

Most studies of scheduling algorithms have focused on optimizing the long term average throughput of the users. The goal is to find algorithms for scheduling the transmissions such that the queues are stabilized at given offered load. An important result along this direction is the development of the so called "throughput - optimal "algorithms [3]. A scheduling algorithm is called throughput – optimal if, at any offered load under which any other algorithm can stabilize the system, this algorithm can stabilize the system as well.

Unfortunately calculating the exact queue distribution is often mathematically intractable. In this paper, we use large deviation theory and reformulate the QoS constraint in terms of the asymptotic decay rate of the queue – overflow probability as threshold value approaches infinity.

II. Background Knowledge

A. Related Research

1.. V.J. venkataraman and X.Lin , " Structural properties of LDP for queue - length based wireless scheduling algorithms" in proc. 45th annu, alierton conf. commun. control, comput., monticello, il, sep. 2007.

Authors proposes scheduling algorithms that can stabilize the network at given offered loads, which also ensures that the long term average, service rate is no less than the arrival rate of each user. Then they describes the class of queue length based scheduling algorithms will have a lower queue overflow probability compared to the queue unaware algorithms because they make an effort to suppress longer queues.

They assumed that a sample path large deviation principle holds for the backlog process, then they first establish a structural property of the minimum cost path to overflow for the class of α – algorithms. Using this structural property, they show that when α approaches infinity, the α -algorithms asymptotically achieve the largest decay rate of the queue-overflow probability.

Finally, their result enables them to design scheduling algorithms that are both close to optimal in terms of the asymptotic decay rate of the overflow probability and empirically shown to maintain small queue-overflow probabilities over queue-length ranges of practical interest.

2. X.Lin, N.B. Shroff, and R.Srikant , " A tutorial on cross – layer optimization in wireless networks", IEEE J.Sel. Areas commun., vol.24, no 8, pp. 1452 – 1463, Aug. 2006.

This tutorial paper overviews recent developments in optimization based approaches for resource allocation problems in wireless systems. The authors begin by over viewing important results in the area of channel aware scheduling for cellular networks, where easily implementable myopic policies are shown to optimize system performance. They demonstrate how to use imperfect scheduling in the cross layer framework and describe recently developed distributed algorithms along these lines.

They describe the use of optimization approach for two classes of cross - layer problems, namely, the opportunistic scheduling problem in cellular, and the joint congestion – control and scheduling problem in multi-hop wireless networks.

3 .L.Tassiulas and A.Ephremides , "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," IEEE Trans. Autom. Control, Vol 37, no 12, pp. 1936 – 1948, Dec. 1992

The authors describe the dependency of servers by the definition of their subsets that can be activated simultaneously. They describe the problem of scheduling the server activation under the constraints imposed by the dependency among them. They also describe the behaviour of the network for arrival rates that lie outside the stability region.

They consider the queuing network model that is suitable for communication network with interdependent service components. Their main motivation for the consideration of this constrained queuing network model is to study the resource allocation problem in multihop radio networks.

4. D.Wu and R.Negi, "Utilizingn multiuser diversity for efficient support of quality of service over a fading channel ", IEEE Trans., Veh. Technol., vol 54, no 3, pp. 1198 – 1206, May 2005.

The authors consider the problem of quality of service provisioning for K users sharing a downlink time slotted fading channel. Then they develop simple and efficient schemes for admission control, resource allocation, and scheduling, which can yield substantial capacity gain.

A unique feature of their work is explicit provisioning of statistical QoS, which is characterized by a data rate, delay bound and delay – bound violation probability. Their approach substantially increases the statistical delay – constrained capacity of a fading channel, when delay requirements are not very tight.

Thus the paper combines crucial ideas from the areas of communication theory and queuing theory to provide the tools to increase capacity and yet satisfy QoS constraints

5. D.Bertsimas, IC.Paschalidis, and J.N.Tsitsiklis, "Asymptotic buffer overflow probabilities in multiclass multiplexers: An optimal control approach, "IEEE Trans. Autiom. Control, vol 43, pp. 315 – 335, Mar.1998

In this paper, the authors propose a multiclass multiplexer with support or multiple service classes and dedicated buffers for each service class. They assume dependent arrival and service processes as is usually the case in models of busty traffic. They describe lower and a matching upper bound on the buffer overflow probabilities.

The authors focus on a simplified version of the problem which retains the most salient features that is, it is multiclass and has correlated arrival and service process. The optimal control formulation provides particular insight into the problem, as it yields an explicit and detailed characterization of the most likely modes of overflow. They relate the lower bound derivation to a deterministic optimal control problem, which they explicitly solve. They specialize their results to the generalized processor sharing policy and the generalized longest queue first policy.

III. α – Algorithm Mechanism

A. System Overview

Consider a downlink of a single cell in a cellular network. The base station transmits to N users. Assume a slotted system, the state of the channel at each time slot is chosen form one M possible states. Let C(t) denote the state of the channel at time t= 1,2,……., There is a queue Q_i associated with each user i=1,2, …. N. Due to interference, at any given time, the base station can only serve the queue of one user. Hence this system can be modelled as a single server serving N queues.

B. System Architecture

Assume that data for user i arrive as fluid at a constant rate. Let $Q_i(t)$ denote the backlog of user is at time t, and let $Q(t) = [\, Q_1(t) , Q_2(t) \ldots Q_N(t) \,]$.

In general, the decision of picking which user to serve is a function of the global backlog Q (t) and the channel state C (t). Let U (t) denote the index of the user picked for service at time.

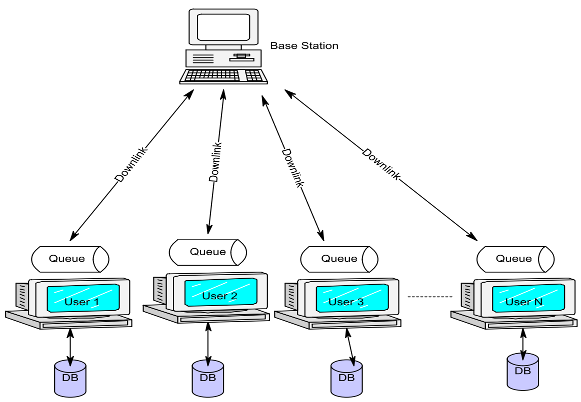

Fig. 1: System architecture

C. *System Algorithm*

First derive an upper bound on the decay rate of the queue overflow probability over all scheduling policies, then focus on a class of scheduling algorithms collectively referred to as the " α – algorithm ". α is a parameter that takes values from the set of natural numbers.

Given α, the behaviour of the algorithm is as follows:

1. Algorithm Steps:

- When the backlog of the user is Q (t) and the state of the channel is C (t) = m, the algorithm chooses to serve the user i for which the product $Q_i^{\alpha}(t) F^{i}_{m}$ is the largest.
- If there are several users that achieve the largest $Q_i^{\alpha}(t) F^{i}_{m}$ together, one of them is chosen arbitrarily.
- It is well known that this class of algorithms is throughput optimal, i.e. they can stabilize the system at the larger set of offer loads [3].
- These algorithms do not explicitly keep track of past history, they do so implicitly by their dependence on Q (t).
- Hence they are able to stabilize the system without explicit knowledge of the operating conditions such as arrival rate and channel probabilities.

The main results are as follows:

for any scheduling algorithms,

$$\liminf_{B\to\infty} \frac{1}{B}\log(P[\max Q(0) \geq B]) \geq -I_{opt} \qquad (1)$$

Where B is the optimal threshold,

I_{opt} is the optimal decay rate.

for α – algorithms,

$$\lim_{\alpha\to\infty}\ \limsup_{B\to\infty} \frac{1}{B}\log(P[\max Q_i(T) \geq B]) \geq -I_{opt} \qquad (2)$$

Where P is the probability measure for the α – algorithm.

Hence, when α approaches infinity, the α – algorithms asymptotically achieve the largest decay rate of the queue overflow probability.

IV. Large Deviation Theory

The problem of calculating the exact probability is often mathematically intractable. In this paper, focus on large deviation theory to compute estimate of this probability. Large deviation theory has been successfully applied to wire-line networks and to wireless scheduling algorithms that only use the channel state to make the scheduling decisions [4]. However, when applied to wireless scheduling algorithms that also use the queue – length to make scheduling decisions (e.g. the α – algorithms), this approach encounters a significant amount of technical difficulty.

Specifically, in order to apply the large deviation theory to queue - length -based scheduling algorithms, one has to use sample – path large deviation and formulate the problem as a multidimensional calculus – of – variations (CoV) problem for finding the "most likely path to overflow". The decay rate of the queue – overflow probability then corresponds to the cost of this path, which is referred to as the "minimum cost to overflow".

A. Lyapunov Function.

Unfortunately, for many queue- length- based scheduling algorithms of interest, this multidimensional calculus – of – variations problem is very difficult to solve. Only some restricted cases have been solved. In the literature, either restricted problem structures are assumed. E.g. symmetric users and ON – OFF channels or the size of the system is very small (only two users).

In this paper, to circumvent the difficulty of the multidimensional CoV problem, we use a Lyapunov function to map the multidimensional CoV problem to a one – dimensional problem, which allows us to bound the minimum cost to overflow by solutions of simple vector optimization problems.

The advantage of working with the α – algorithms instead of the exponential rule is that the α – algorithms are scale – invariant (i.e, the outcome of the scheduling decision does not change if all queue lengths are multiplied by a common factor). Hence, it can use the standard sample- path large – deviation principle (LDP) instead of the refined LDP. The results highlight the role that the exponent α plays in determining the asymptotic decay rate.

Finally, using the insight of the main result, can design a scheduling algorithm that is both close to optimal in terms of asymptotic decay rate of the overflow probability and empirically shown to maintain small queue - overflow probabilities over queue - length ranges of practical interest.

V. Conclusion

In this paper, to focus on the class of "α – algorithms", which pick the user for service at each time that has the largest product of the transmission rate multiplied by the backlog raised to the power of α. When α approaches infinity, the α – algorithms asymptotically achieve the largest decay rate f the queue overflow probability. A key step in proving this result is to use Lyapunov function to derive an simple lower bound for the minimum cost to overflow under the α – algorithms. This technique, which is of independent interest, circumvents solving multidimensional calculus – of – variations problem typical in this type of problem.

Finally , using the insight from this result , can design hybrid scheduling algorithms that are both close to optimal in terms of the asymptotic decay rate of the overflow probability and empirically shown to maintain small queue – overflow probabilities over queue – length ranges of practical interest.

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