IJCAES
ISSN: 2231-4946
Volume III, Special Issue, August 2013

International Journal of Computer Applications in Engineering Sciences
Special Issue on National Conference on Information and Communication (NCIC'13)
www.caesjournals.org

Cramer-Rao Bound For Offset Timing Estimation Of Ofdm Systems

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***Abstract*— In an orthogonal-frequency-division multiplex (OFDM) system, the problem of sampling clock synchronization and channel estimation is analyzed. In such systems, when the number of sub-carriers is large, a sampling clock frequency mismatch between the transmitter and the receiver dramatically degrades the performance. Obviously, the channel-impulse response is unknown at the receiver and also needs to be estimated. Therefore, the CRB associated with this joint estimation is theoretically evaluated. When the number of sub-carriers and the channel degree are large, very compact closed-form expressions for the CRB are obtained. Furthermore, along with the maximum-likelihood (ML) estimator, sub-optimal estimation algorithms are introduced and compared with some existing approaches and with the CRB.**

***Keywords*—Channel estimation, Cramer–Rao bound (CRB),Orthogonal-frequency-division-multiplex (OFDM), sampling clock offset estimation, very high speed digital subscriber line (VDSL).**

I. INTRODUCTION

The detection of orthogonal-frequency-division- multiplexing (OFDM) symbols cannot be done properly without a reliable clock synchronization One synchronization step consists of estimating the OFDM symbol timing, which is the delay between the transmitted and the received OFDM symbols. In a certain number of applications where these symbols are short, estimating this delay is enough. However, as soon as the number of samples per OFDM symbol (or equivalently, the number of sub-carriers) becomes large, the frequency offset between the transmitter's sampling clock and the receiver's sampling clock in its free oscillation mode has to be considered too. Indeed, this offset leads to a sampling delay that drifts linearly in time over the OFDM symbol. Without any compensation, this drift hampers the receiver's performance as soon as the product of the relative clock frequency offset with the number of sub-carriers becomes non negligible . For instance, in very high speed digital subscriber line (VDSL) transmissions, these two quantities can reach 10^{-4} and 4096 respectively, making the clock frequency offset compensation mandatory. As it is well known, the part of the OFDM symbol that enters the fast Fourier transform (FFT) device at the receiver comes after a cyclic prefix. As the latter has a length comparable to the channel impulse response length, it is precisely when the channel is long that a long duration has to be chosen for the useful part of the OFDM symbol, in order to reduce the impact of the cyclic prefix on the spectral efficiency. It is therefore worth considering the problem of the joint estimation of the clock frequency offset and of the channel-impulse response, particularly in these situations where the observation window has to be rather large. The literature proposes several data-aided algorithms (in the sense that one or several OFDM symbols are devoted to training) to perform the estimation of the clock frequency offset. In some of these approaches, the channel is implicitly assumed perfectly known while in others, the knowledge of the channel is not required to perform the frequency offset estimation. In this paper, in order to better understand the interactions between these two estimations, we begin in Section III by giving the Cramér–Rao bound (CRB) associated with this joint estimation problem. This is, in particular, the case of VDSL, whose degree is often of the order 100. Section IV deals with practical estimation algorithms. We begin by the maximum-likelihood (ML) estimator for which we propose a simplified version. Because the ML algorithm remains complicated even in its simplified version, we study simple estimation algorithms that require OFDM training symbols having particular structures. In Section V, the ML algorithm as well as sub-optimal algorithms are tested and compared with the CRBs. Concluding remarks are drawn in Section VI.

II. SYSTEM MODEL

The continuous-time received signal $y_a(t)$ writes as follows :

$$y_a(t) = \sum_{k \in Z} d_k g_a(t - kT) + b_a(t) \qquad (1)$$

where $\{d_k\}_{k\epsilon z}$ is the training sequence known to the receiver. The unknown impulse response $g_a(t)$ represents the complete channel that includes the transmit filter, the propagation channel, and the receiver low-pass filter. Throughout the paper, we assume that the filter $g_a(t)$ is time-limited and causal. Although unbounded in theory, the frequency support of $g_a(t)$ will in practice be assumed to coincide with $[-\rho /2T,\rho/2T]$ for some known parameter ρ. Finally $b_a(t)$ is an additive noise independent of the data. Equation above models a large number of digital signals : in the case of standard single carrier modulated signals, $\{d_k\}_{k\epsilon z}$ are the training symbols, T is the symbol rate, and ρ coincides with 1 + α where α is the roll-off factor. In a multicarrier OFDM setting, $\{d_k\}_{k\epsilon z}$ represents the output of the IFFT device, T is the sampling period, and $\rho \leq 1$. If the transmitter and receiver clocks were perfectly synchronized, then the signal $y_a(t)$ would have been sampled at the period $T_s = T/q$, where $q \geq 1$ is an integer "over sampling factor" satisfying the conditions of the sampling theorem. In the absence of synchronization, $y_a(t)$ is sampled at $(1 + \delta)T_s$ instead of T_s, where δ is an unknown offset lying in the known interval $[-\delta_{max}, \delta_{max}]$. The discrete-time signal $y(n) = ya(n(1 + \delta)T_s)$ thus writes

$$y(n) = \sum_{k \in Z} d_k g_a(nT_s - kT + n\delta T_s) + b(n) \qquad (2)$$

In the sequel, $b(n) = b_a((1 + \delta)T_s)$ is assumed white Gaussian circular with zero-mean and known variance $\sigma^2 = E[|b(n)|^2]$.

Let $\{s_k\}_{k\epsilon z}$ be the sequence defined as $s_k = d_{k/q}$ if $k/q \in Z$, and $s_k = 0$ otherwise. The previous equation then takes the following form

$$y(n) = \sum_{k \in Z} s_k g_a((n - k + n\delta)T_s) + b(n) \qquad (3)$$

Because $g_a(t)$ is band-limited, it can be written as

$$g_a(t) = \sum_{l \in Z} h_l p_a(t - lT_s) \qquad (4)$$

where $p_a(t)$ is any interpolation filter with /2T as a cut-off frequency. As $g_a(t)$ is time-limited, $p_a(t)$ is assumed time supported by $[0,MT_s]$ and the sequence $\{h_l\}_{k\epsilon z}$ is assumed finite with length L+ 1. The paper focuses on the joint estimation of the sampling clock offset δ and of the channel impulse response $g = [g_0,\ldots,g_{L+M}]^T$ with $g_l = g_a(lT_s)$. Clearly the interpolator $p_a(t)$ is available at the receiver while the vector

$h = [h_0,\ldots,h_L]^T$ is not.

III. EXACT CRAMER-RAO BOUND

NT_S being duration of the observation window, we have

$$y(n) = \sum_{k=-KN}^{KN=M} \sum_{l=o}^{L} h_l s_{n-k-l}\, p(k + n\delta) + b(n) \qquad (5)$$

where p(u) = p_a(uTs) and KN is the smallest integer such that KN ≥ Nδmax. The aim of the section is to derive CRB on $[g^{-T}, \delta]^T$ where $\tilde{g}$ =$[\Re[g^T],\Im[g^T]]^T$.Notations $\tilde{g}$ =ℜ[.] andℑ[.] stand for real and imaginary parts of a complex-valued matrix respectively.By setting $\mathbf{y}_N = [y(0); : : : ; y(N - 1)]^T$, equation (2) can be put in a matrix form :

$$y_N = S_N PN(\delta)h + bN \qquad (6)$$

where

$$s_N = \begin{bmatrix} s_0 & 0 \\ 0 & s_{N-1} \end{bmatrix}, P_N(\delta) = \begin{bmatrix} P(0) \\ . \\ . \\ . \\ P((N-1)\delta) \end{bmatrix}$$

With

$$S_n = [S_n + K_n, \ldots, S_n - (M + K_N) - L]$$

and with p(nδ) being the (2KN +M+L+1)*(L+1)

Toeplitz matrix having [p(nδ - KN), 0,…, 0] as its first row and $[p(n\delta-KN),..,p(n\delta+KN+M), 0,\ldots, 0]^T$ as its first column. The vector y_N is circular multivariate Gaussian process with unknown mean $\mu = S_N P_N(\delta)h$ and known covariance matrix $\sigma^2\ \mathbf{I}_N$. At this point, we need the expression of the Fisher Information Matrix (FIM) on the parameter vector

$\theta = [\theta_{0,\ldots,}\theta_{2(L+1)}]^T = [\tilde{h}^{-T,}\delta]^T$ where $\tilde{h} = \Re[h^T]$ and $\Im[h^T]$.The FIM expresses as follows

$$F_N = \frac{2}{\sigma^2}\Re\left[\frac{\partial\mu}{\partial\theta}^H \cdot \frac{\partial\mu}{\partial\theta}\right]$$

$$\frac{\partial\mu}{\partial\theta} = \left[\frac{\partial\mu}{\partial\theta_0}, \ldots, \frac{\partial\mu}{\partial\theta_2(L+1)}\right]$$

$$F_N = \frac{2}{\sigma^2}\begin{bmatrix} N\Re[U_N] & -N\Im[U_N] & N^2\Re[V_N h] \\ N\Im[U_N] & N\Re[U_N] & N^2\Im[V_N h] \\ N^2\Re[h^H V_N^H] & -N^2\Im[h^H V_N^H] & N^3 h^H W_N h] \end{bmatrix}$$

where

$$U_N = \frac{1}{N} P_N^T(\delta) S_N^H S_N P_N(\delta)$$

$$V_N = \frac{1}{N^2} P_N^T(\delta) S_N^H S_N Q_N(\delta)$$

$$W_N = \frac{1}{N^3} Q_N^T(\delta) S_N^H S_N Q_N(\delta)$$

with QN(δ) = dPN(δ)=d/dδ. The reason for introducing the factors 1/N, $1/N^2$, and $1=N^3$ will become apparent in the next paragraph. By applying the well known formulas for the inversion of block partitioned matrices, we obtain :

$$F_N^{-1} = \begin{bmatrix} A_N & b_N \\ b_N^T & c_N \end{bmatrix}$$

where

$$A = \frac{\sigma}{2N}\left(\begin{bmatrix} \Re[U_N^{-1}] & -\Im[U_N^{-1}] \\ \Im[U_N^{-1}] & \Re[U_N^{-1}] \end{bmatrix} + \frac{1}{\gamma N}\begin{bmatrix} \Re[\beta_N] \\ \Im[\beta_N] \end{bmatrix}\begin{bmatrix} \Re[\beta_N^T] & \Im[\beta_N^T] \end{bmatrix}\right)$$

$$b_N = \frac{-\sigma^2}{2N^2\gamma N}\begin{bmatrix} \Re[\beta_N] \\ \Im[\beta_N] \end{bmatrix}$$

$$C_N = \frac{\sigma^2}{2N^3\gamma N}$$

$$\beta_N = U_N^{-1}V_N h,$$

$$\gamma_N = h^H (W_N - V_N^H U_N^{-1} V_N) h$$

One can notice that g = Ph where P is the (L+M+1)*(L+1) Toeplitz matrix with first row [p(0),…,0] and first column [p(0),…, p(M),0,.., 0]^T. Therefore the CRB on [g$^{-T,}$ δ]^T expresses as
Follows

$$\begin{bmatrix} CRB_N^{g,g} & CRB_N^{g,\delta} \\ CRB_N^{\delta,g} & CRB_N^{\delta,\delta} \end{bmatrix} = \begin{bmatrix} \widetilde{P} A_N \widetilde{P}^T & \widetilde{P} b_N \\ b_N^T \widetilde{P}^T & C_N \end{bmatrix}$$

where $\widetilde{b}$ = [p,0;0,p]. In particular, we have

$$E\left[\|\hat{g}N - g\|^2\right] \geq \frac{\sigma^2}{2N}\left(2tr(PU_N^{-1}P^T) + \frac{\beta_N^H P^T P \beta_N}{\gamma N}\right)$$

IV. ESTIMATION ALGORITHMS

A. ML-Like Algorithms

Getting back to the received signal in the frequency domain, the log-likelihood function to be minimized L (θ) is

$$L(\theta) = \left\| Y_N - F_{N,N} R_N(\delta) g \right\|^2 \qquad (7)$$

The minimization of L(θ) leads to the following ML-based estimates δ of and h_N:

$$\hat{\delta}_N = \arg\max_{\delta} Y_N^H \prod_N(\delta) Y_N \qquad \hat{g}_N = (R_N^H(\hat{\delta}_N) R_N(\hat{\delta}_N))^{-1} R_N^H(\hat{\delta}_N) F_{N,N}^H Y_N$$

$$\hat{h}_N = E_N \hat{g}_N$$

where

$\prod_N(\delta) = F_{N,N}(R_N^H(\delta)R_N(\delta))^{-1} R_N^H(\delta_N) F_{N,N}^H$ is the orthogonal projection matrix onto the subspace of C^N spanned by the columns of $F_{N,N}(R_N(\delta)$. For estimating the sampling clock offset, each try of a value of δ requires the inversionof $R_N^H(\delta)R_N(\delta)$.The implementation of this algorithm is therefore impractical. However, it can be simplified in the asymptotic regime described at the end of the previous section. In this regime, can indeed be replaced with

$$\underline{\prod}_N(\delta) = \frac{1}{N} F_{N,N} R_N(\delta) U^{\#} R_N^H(\delta) F_{N,N}^H \quad (8)$$

Notice that $U^{\#}$ is independent of δ and so this matrix is computed only once. Notice also that, because we are only able to consider the significant eigenvalues of U,we can only estimate for. Nevertheless, as the parameter of interest is h_N , values of G(e $^{2if\pi}$)out of this set are not needed, and therefore the estimate remains accurate. Here, the estimation $\hat{h}_N$ algorithm becomes

$$\hat{\delta}_N = \arg\max_{\delta} Y_N^H \prod_N(\delta) Y_N \qquad (9)$$

$$\hat{h}_N = \frac{1}{N} E_N U^{\#} R_N^H(\tilde{\delta}_N) F_{N,N}^H Y_N \qquad (10)$$

B. Suboptimal Algorithms

The complexity of the ML algorithm presented in the previous subsection prevents its implementation in most practical situations even if one resorts to the simplification (10). It appears that the estimation problem can be largely simplified by endowing the OFDM training symbol with a particular structure. The principle of the approach is the following. Neglecting the additive noise, let us assume that the received sequence (y_N (0),…,y_N (N-1)consists of two identical parts of length N/2 each, i.e., y_N (n) = y_N (n+N/2) for n=0,..,N/2-1. This comes down to setting the N/2 symbols at the odd subcarriers to zero in the transmitted OFDM symbol, or in other words, the training sequence ($D_{N,0,\dots,}$ $D_{N,N-1}$) in the frequency domain is asserted to satisfy $D_{N,2l+1}$ =0 for l= 0,..,N/2 - 1. At the receiver side, two consecutive FFTs with length N/2each are performed. If δ were equal to 0, then the outputs of these FFTs would be identical. When δ ≠ 0, if we neglect the so-called intercarrier interference (ICI) created by this mis-synchronization, then the m^{th} output of the second FFT is equal to the m^{th} output of the first FFT rotated by the angle 2πδm. The delay δ can thus be estimated from these rotations. The idea of transmitting two identical signal halves and exploiting this structure for synchronization is not new. It appeared in the context of Doppler shift estimation. Notice that when a Doppler shift exists, the output of the second FFT is equal to the output of the first FFT rotated by a constant angle (instead of in the sampling clock offset estimation context).

The least-square (LS) estimate of the channel writes

$$\hat{\delta} = \sqrt{\frac{2}{M}} \left(\underline{E}_M^H \underline{X}_M^H \underline{\theta}_M^H(\hat{\delta}) \underline{\psi}_M^H(\hat{\delta}) \underline{\psi}_M(\hat{\delta}) \underline{\theta}_M(\hat{\delta}) \underline{X}_M \underline{E}_M \right)^{-1} \times \underline{E}_M^H \underline{X}_M^H \underline{\theta}_M^H(\hat{\delta}) \underline{\psi}_M^H(\hat{\delta}) \tilde{Y}_{N.} \qquad (11)$$

The inversion operation in this equation increases dramatically the implementation complexity. For this reason, the simpler estimate can be used instead

$$\hat{\delta} = \frac{1}{2\sqrt{2M}} \underline{E}_M^H \underline{X}_M^H \underline{\theta}_M^H(\tilde{\delta}) \underline{\psi}_M^H(\tilde{\delta}) \tilde{Y}_N \qquad (12)$$

V. SIMULATION RESULTS

Simulations are carried out in a single carrier context. The training sequence $\{d_k\}$ is a QPSK constellation based pseudo-random white sequence. The magnitude of the channel transfer function used in this section is represented on Fig. 1. The corresponding channel impulse response sampled at 20 MHz is made up of 80 complex coefficients. Via computer simulations, we compare the performance of the Estimation Algorithms suboptimal methods introduced in Section IV. In Fig. 2, Channel estimation of AWGN channel , the mean square errors of the ML and suboptimal estimates. Here, N varies from 200 to 5000 and Signal to Noise Ratio(SNR) is 20 dB. In fig. 3, Symbol timing error estimation of AWGN channel.

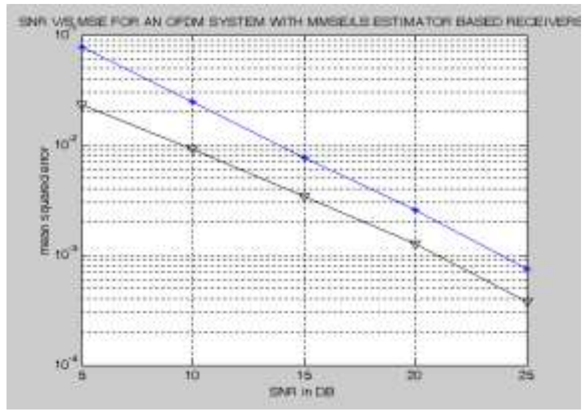

Fig. 1Channel estimation of AWGN channel

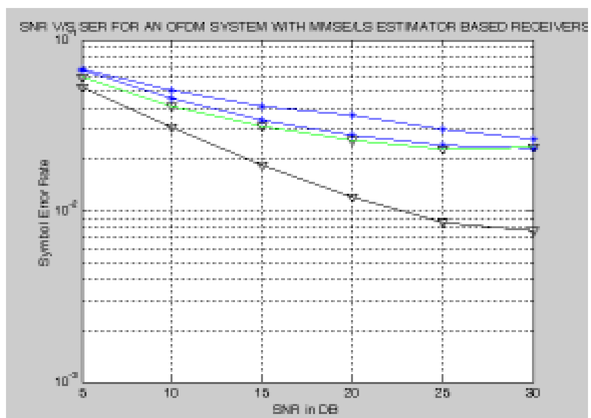

Fig.2 Symbol timing error estimation of AWGN

VI. CONCLUSION

In this paper, we considered the issue of joint sampling clock offset estimation and channel estimation for OFDM System. The estimation performance has been studied through the CRB analysis. The ML joint algorithm has been derived. Suboptimal approaches have also been proposed and compared with ML algorithm.

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